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LETTER TO THE EDITOR

Spreading of 'damage' in a three-dimensional Ising model

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Abstract. We study the time evolution of two three-dimensional Ising configurations by Monte Carlo simulation, which differ only by the central spin. By monitoring the spreading of the damaged spins, we observe a phase transition at a value of the temperature in agreement with the Ising-correlated site percolation.

The study of the dynamics of very complicated systems, like growth models or energy transport in cellular automata, is a field of great current interest [1, 2]. In particular, Stanley *et al* [3] have recently investigated the spreading of a small perturbation in two-dimensional Ising models and in Q2R cellular automata. They find that for both models spreading is hindered below a threshold temperature which agrees numerically with the Curie point. In order to answer the question: 'What occurs in a more cooperative system?' we investigate in this letter the same problem for the three-dimensional Ising model. This investigation may also clarify some aspects of more

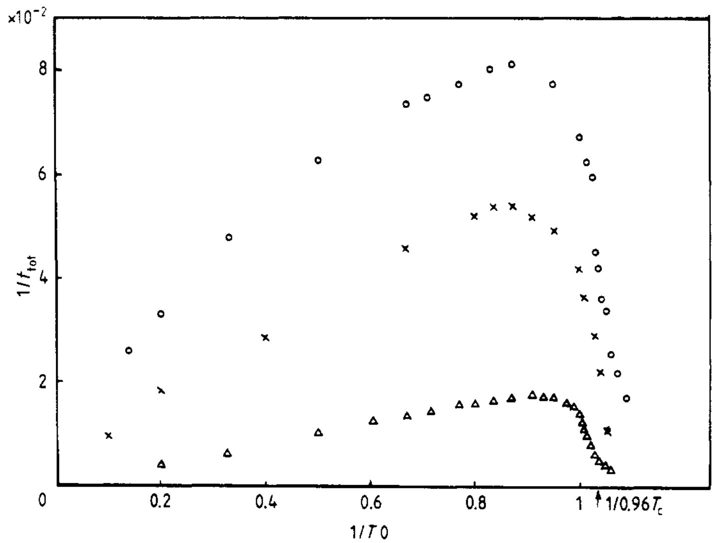

Figure 1. Inverse time ($1/t_{tot}$) for all sites to be damaged at least once against inverse temperature ($1/T0 = 1/T^* T_c$), for $L = 10$ (○), $L = 25$ (+) and $L = 40$ (△).

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complicated three-dimensional problems, like the spreading of a single mutation in a genetic system or as an isolated hardware failure affects the performance of a big computer [3-5].

We consider a three-dimensional system of Ising spins on simple cubic lattice A and we let the system reach equilibrium at a given temperature. We then take another lattice B, equal to the lattice A, and we flip only the central spin in lattice B. Finally, we study the subsequent time evolution of this defect and for each time step we simulate the two lattices A and B with the same algorithm and the same random number and determine the damage, i.e. the number of spins which differ between the two configurations A and B. This procedure is repeated for several temperatures and lattice sizes.

The standard Metropolis algorithm with periodic boundary conditions is used to determine the spin configurations for small lattices; the size L is increased from 5 to 25, and for $L = 40$ we utilise multispin coding techniques and helical boundary conditions (for a review of these Monte Carlo methods see ch 1 of [6]).

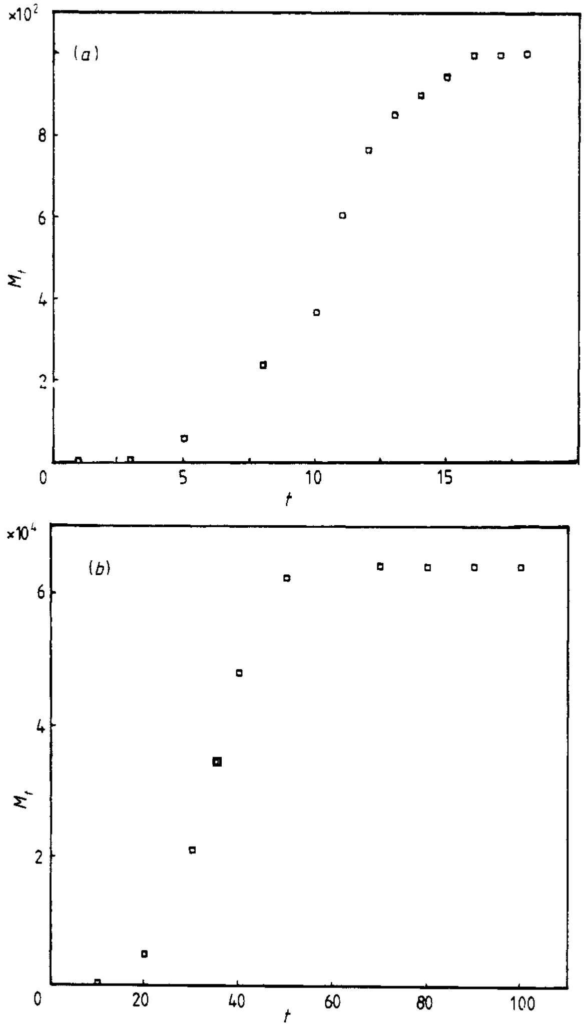

Figure 2. Average number of sites damaged (M_t) against time (t) for $T = 1.5T_c$. (a) $L = 10$, (b) $L = 40$.

We expect that the average magnetisation will not change by introducing such a small perturbation. Therefore we consider as 'order parameter' or damage the number of spins which differ in the two configurations A and B and we calculate this damage as a function of the temperature and of the lattice size and the following quantities:

(i) M_t, the number of spins in B that are different from the corresponding spins in A,

(ii) t_{tot}, the time for all sites of B to be damaged at least once, and

(iii) t_t, the time for the damage to hit the upper or lower boundary of the lattice.

As we can see from figure 1 there is a threshold spreading temperature below which the damage does not spread anymore, as $t \to \infty$. Thus $t_{tot} = \infty$ below that temperature. This temperature does not agree with the Curie critical temperature, but rather with the critical temperature of the Ising-correlated site percolation [7], which may mean that the problem of spreading in the Ising model is in some sense 'comparable' with the Ising-correlated site percolation problem. Stanley *et al* [3] could not observe this effect because in two dimensions the critical point for Ising-correlated site percolation and the Curie critical temperature for the Ising model are the same. The change in slope of this curve represents the finite-size effect.

It is interesting to remark that, even if we observe the same behaviour for three- and two-dimensional systems, for the two-dimensional system the average time needed to damage every spin in the whole sample at least once is bigger than for the three-dimensional system (about four times greater for lattices $L = 40$ and the maximum occurs at different temperatures). This is not surprising because, since we have more correlation in the three-dimensional lattices, the damage propagates faster. Here we can ask the following question: what happens if we take the two-dimensional or the three-dimensional Ising model with second- and third-neighbour interactions? We would expect that the time for all the sites to be damaged at least once will be less than in the case of first-neighbour interactions, i.e. the damage will propagate more rapidly and that the spreading temperature differs even more from T_c [8].

In figures 2(a) and 2(b) we present the number of sites damaged (M_t) as a function of the time for $T = 1.5T_c$ (where T_c is the Curie temperature) and for L equal to 10

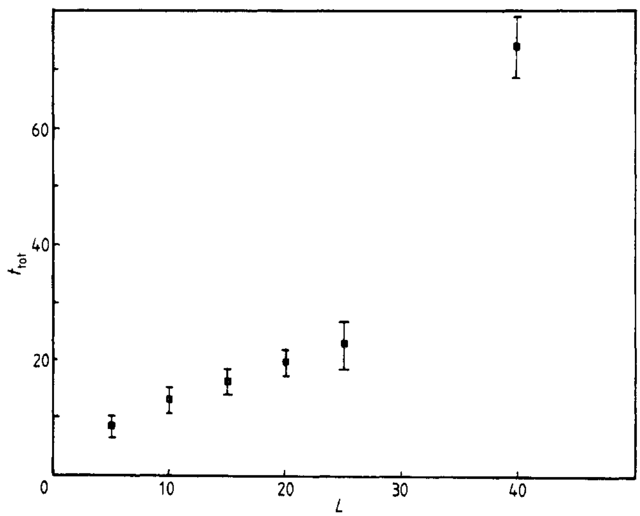

Figure 3. Average time for all the spins to be damaged at least once (t_{tot}) against the lattice size (L) at $T = 1.5T_c$.

and 40. The same behaviour is observed for both sizes. Finally, in figure 3 we show the average time for all the spins to be damaged at least once (t_{tot}) as a function of the size of the lattice.

In conclusion, we have calculated via computer simulation the time evolution of two three-dimensional Ising configurations which differ by only one spin at time $t = 0$. We have found that for T below $T^* \approx 0.96 T_c$ the damage goes to zero and this temperature is in agreement with the critical temperature for the Ising-correlated site percolation. These two phenomena are indeed different: one is a dynamical phenomenon and the other one is a static phenomenon, and so having found the same temperature for both phenomena could be a simple coincidence. Some work in order to clarify this question would be welcome.

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Note added in proof. Later simulations for lattice size $L = 120$ found the same spreading temperature.

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